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Antiproton-deuteron annihilation at low energies

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Abstract. Recent experimental studies of the antiproton-deuteron system at low energies have shown that the imaginary part of the antiproton-deuteron scattering length is smaller than the antiproton-proton one. Two- and three-body systems with strong annihilation are investigated and a mechanism explaining this unexpected relation between the imaginary parts of the scattering lengths is proposed.

PACS. 25.43.+t Antiproton-induced reactions

1 Introduction

Recent measurements of antiproton-deuterium and antiproton-helium annihilation cross-sections at low antiproton momentum ($p_{\rm lab}$ down to 35 MeV/c) [1] and of the shift and the width of the antiproton-deuteron atomic ground state [2] have provided the first information on the interaction of slow antiprotons with light nuclei. Both experiments gave unexpected but coherent results. The width of the $1S$ state of the $\bar{\rm p}$D atom appears to be approximately equal to that for protonium. In the region of low antiproton momentum, the annihilation cross-sections in the $\bar{\rm p}$p, $\bar{\rm p}$D and $\bar{\rm p}^4$He systems are approximately equal.

The phenomenological analysis of these data [3] allows the imaginary parts of the S-wave scattering lengths for the three nuclei to be extracted. The values was found to decrease with atomic number

$$\begin{aligned}
\text{Im } a_{\rm sc}(\bar{\rm p}{\rm p}) &= -[0.69 \pm 0.01(\text{stat}) \pm 0.03(\text{sys})] \text{ fm},\\
\text{Im } a_{\rm sc}(\bar{\rm p}{\rm D}) &= -[0.62 \pm 0.02(\text{stat}) \pm 0.05(\text{sys})] \text{ fm},\\
\text{Im } a_{\rm sc}(\bar{\rm p}^4{\rm He}) &= -[0.36 \pm 0.03(\text{stat})^{+0.19}_{-0.11}(\text{sys})] \text{ fm}.
\end{aligned}$$

This result is in direct contradiction with the naive geometrical picture of annihilation which would suggest that the imaginary part of the scattering length (or corresponding width and cross-section) increases with the size of the nucleus.

Only a few calculations of the atomic $\bar{\rm p}$D system [4–6] have been carried out prior to the measurements. In the first one [4], the authors used an approximate iterative method to obtain the optical $\bar{\rm p}$D potential starting from elementary $\bar{\rm N}$N and NN interactions. This method was applied previously to atomic data for heavier nuclei [7,8]. In the second calculation [5], the $\bar{\rm p}$D system interaction was studied by solution of the three-body equations. In both approaches, the elementary $\bar{\rm N}$N and NN interactions was chosen in separable form (this limits the conclusions due to particular properties of the separable interaction). In the two articles, the same numerical result was obtained: the width of the $1S$ $\bar{\rm p}$D atomic level appeared to be smaller than that for the $\bar{\rm p}$p system and the necessity to push the calculations beyond the simple impulse approximation was emphasized. The same conclusion was obtained in [9]. In a third article [6], the authors have investigated mostly different approximate solutions of the three-body problem. Due to the strong annihilation the results was found to be not very sensitive to the three-body dynamics. However, the shifts and the widths of the $1S$ $\bar{\rm p}$D atomic level obtained in this article are quite different from the ones observed experimentally.

The aim of the present work is to understand why the simple geometrical picture does not work and to propose a mechanism, at least for $\bar{\rm p}$D system, to explain the unexpected behavior of the scattering length.

Our explanation is based on two points. The first one is the fact that the imaginary part of the scattering length for the $\bar{\rm N}$N optical potentials with strong annihilation is mainly determined by the diffuseness of the exponential tail of the potential, but not by the details of the inner part of the interaction. Therefore, one can expect that Im $a_{\rm sc}(\bar{\rm p}{\rm p})$ and Im $a_{\rm sc}(\bar{\rm p}{\rm D})$ can be, at least, close to each other, in spite of the different structure of interactions in $\bar{\rm p}$p and in $\bar{\rm p}$D systems. This expectation is confirmed by our three-body calculation of Im $a_{\rm sc}(\bar{\rm p}{\rm D})$. The second one is the increase of Im $a_{\rm sc}(\bar{\rm p}{\rm p})$ relative to Im $a_{\rm sc}(\bar{\rm p}{\rm D})$ due to a resonance in the $\bar{\rm N}$N system.

We shall be concerned with general features of annihilation in the $\bar{\rm p}$p and $\bar{\rm p}$D systems which are essential for explaining the observed phenomenon. There are two pos-

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sible methods for taking the annihilation into account: either to introduce the imaginary part to the potential (optical model) or to introduce explicitly effective annihilation channels (coupled-channel model). The first possibility is more simple but leads to some problems related to the non-unitarity of this approach (see [10] and references therein). The second takes the unitarity into account explicitly but the numerical treatment becomes quite involved. Thus, to simplify the calculations only the optical-model approach is considered. Furthermore, a very simple antinucleon-nucleon interaction is used.

To simplify the numerical treatment we will calculate the scattering length $a_{\rm s}$ corresponding to a pure strong interaction. The scattering lengths extracted from the experimental data $a_{\rm sc}$ are modified by the Coulomb interaction. For the $\bar{\rm p}$p system, there is a simple phenomenological relation between $a_{\rm s}$, $a_{\rm sc}$, and the Bohr radius B:

$$\frac{B}{a_{\rm sc}} = \frac{B}{a_{\rm s}} + C\,, \tag{1}$$

with complex constant $C = 7.17 + {\rm i}1.46$ which is valid within 1% accuracy as was demonstrated in [11]. If one supposes that this relation is the same for the $\bar{\rm p}$D system one obtains the following imaginary parts of the scattering lengths:

$$\begin{aligned} {\rm Im}\ a_{\rm s}(\bar{\rm p}{\rm p}) &= -(0.86 \pm 0.04)\ {\rm fm},\\ {\rm Im}\ a_{\rm s}(\bar{\rm p}{\rm D}) &= -(0.80 \pm 0.06)\ {\rm fm}. \end{aligned}$$

The real parts of the scattering lengths are taken from [3], all errors are added quadratically. One can see that in the absence of Coulomb forces the $\bar{\rm p}$D scattering length is still smaller than that for the $\bar{\rm p}$p system.

The article is organized as follows. Section 2 presents the properties of simple two-body interactions in the case of strong annihilation. In sect. 3, the three-body $\bar{\rm p}$D problem is solved numerically by means of Faddeev equations and a mechanism explaining the observed effect is found. Finally, we provide a brief summary of the results.

2 Two-body interactions

Before calculating the $\bar{\rm p}$D scattering length obtained in the next section by the numerical solution of the three-body problem it is instructive to present some analytical results on the scattering length in a few simple two-body optical potentials. These examples emphasize that the diffuseness of the interaction determines the imaginary part of the scattering length. This remains valid for $\bar{\rm p}$D interaction.

For the square-well potential

$$U(r) = \begin{cases} -U_0, & r < R\,,\\ 0, & r > R\,, \end{cases}$$

the scattering length is equal to

$$a_{\rm s} = R\left(1 - \frac{\tan pR}{pR}\right). \tag{2}$$

Here $p^2 = 2mU_0$ with $U_0 \equiv W{\rm e}^{i\varphi}$. In the case of pure imaginary optical potential, $\varphi = \pi/2$. Hereafter, we suppose that $\varphi > 0$. Potentials with an attractive real part have $0 < \varphi < \pi/2$, and repulsive potentials have $\pi/2 < \varphi < \pi$. For a pure imaginary potential, the imaginary part of the scattering length as a function of the imaginary depth starts from zero, reaches a maximum and, in the limit of very strong annihilation ($W \to \infty$), tends to zero as $1/\sqrt{W}$. The scattering length becomes real. This result is well known in physics of hadronic atoms [12]. The scattering on this potential is equivalent to scattering on hard sphere. In this limit, the wave function tends to zero within the region of annihilation. This is the so-called "S-wave suppression" which appears in the optical potential with strong annihilation.

Even in the limit when the range of interaction becomes very large ($R \to \infty$), the imaginary part of the scattering length does not go to infinity:

$$\lim_{R\to\infty} {\rm Im}\ a_{\rm s} = -\frac{1}{\sqrt{2mW}}\cos\frac{\varphi}{2}.$$

The parameter which defines the strength of annihilation is a product $|pR|$ (see (2)). So hereafter, the strong annihilation limit should be taken as implying $|pR| \gg 1$.

The fact that the imaginary part of the scattering length becomes real when the annihilation is strong is related to the sharp edge of the square-well potential. For another analytically solvable potential

$$U(r) = -U_0{\rm e}^{-r/r_0}\,,$$

with $U_0 \equiv W{\rm e}^{i\varphi}$, the scattering length is equal to

$$a_{\rm s} = 2r_0\left[\gamma + \ln z_0 - \frac{\pi}{2}\frac{N_0(2z_0)}{J_0(2z_0)}\right].$$

Here $\gamma = 0.577...$ is Euler constant, $z_0^2 = 2mU_0r_0^2$, $J_0(z)$ and $N_0(z)$ are Bessel and Neumann functions of index 0.

In the limit of strong annihilation ($|z_0| \gg 1$), Im $a_{\rm s}$ is defined by the diffuseness of the interaction r_0:

$$\lim_{|z_0|\to\infty} {\rm Im}\ a_{\rm s} = -r_0(\pi - \varphi).$$

The same asymptotic result for Im $a_{\rm s}$ is valid for any complex potential which has an "exponential tail", for instance, a Woods-Saxon potential which can be considered as a first approximation to describe the antiproton-nucleus atomic data [13]. More sophisticated potentials containing more complicated dependencies are also discussed in the literature (see [14] and references therein).

In the limit of strong annihilation, Im $a_{\rm s}$ depends neither on the range of interaction R nor on its particular form within this region, but it is determined only by the diffuseness of the exponential tail r_0. The wave function Ψ exponentially decreases when r crosses the region of diffuseness of the potential:

$$\Psi \sim \exp\left(\int_0^r \sqrt{2m|U(\rho)|}{\rm d}\rho\right).$$

From the expression for the scattering amplitude

$$f(k) = -\frac{2m}{k^2}\int_0^\infty \sin(kr)\, U(r)\Psi(r)\mathrm{d}r$$

(k being the center-of-mass momentum) one can see that the contribution of the inner part of the interaction is exponentially small due to the behavior of the wave function. Qualitatively this means that the particle in a strong imaginary potential "annihilates" at a large distance and that the probability to penetrate to the origin is exponentially small. The scattering takes place on the exponential tail and is thus determined by r_0.

As was shown in [15], this phenomenon leads to the effect of saturation of the antiproton-nucleus and kaon-nucleus atomic widths.

Another prediction can be made for the value of the antinucleon-nucleus annihilation cross-sections. At very low energies, where the annihilation cross-section σ_a is determined by Im a_s: $k\sigma_\mathrm{a} \approx 4\pi\,|\mathrm{Im}\ a_\mathrm{s}|$, its value does not depend on the atomic number (for nuclei with the same diffuseness). Strictly speaking, this simple conclusion is true for $\bar{\mathrm{n}}$ only (for $\bar{\mathrm{p}}$, the low-energy expression for the annihilation cross-section depends on the real part of the scattering length and contains higher partial-wave parameters [3,16], so that this case needs more careful consideration).

To complete this analysis let us consider an elementary interaction in a separable form $V = g\,|\alpha\rangle\langle\alpha|$ with complex coupling constant g (as chosen in [4,5]). The imaginary part of the amplitude as a function of g has the same qualitative one-bump behavior as the imaginary square-well potential:

$$a_\mathrm{s} = -\frac{g|\alpha(\mathbf{k}=0)|^2}{1 + 2mg\int \mathrm{d}^3\mathbf{k}|\alpha(\mathbf{k})|^2/k^2},$$

where $\alpha(\mathbf{k}) = \langle\mathbf{k}|\alpha\rangle$.

For finite complex g, a_s is complex also. However, for a very strong interaction $|g| \to \infty$, the scattering length becomes real for any form factor $\langle\mathbf{k}|\alpha\rangle$. The reason for such behavior is the "poor" spectral properties of separable interactions: a rank-one separable potential is able to produce only one bound state and all dynamics is governed by the position of the corresponding poles of the S-matrix.

Till now, only potentials which have the same range of the real and of the imaginary part were considered. However, the optical potentials describing elementary antiproton-proton interaction are quite different. For instance, in the case of the popular Kohno-Weise potential [17, 18] which reproduces rather well all data on low-energy nucleon-antinucleon interaction, the range of the real part of the interaction is larger than that of the imaginary part (see fig. 2 in [18]). For these potentials, the existence of an external real part of the potential can significantly modify the situation.

As an example, let us consider the simplest potential composed of two square wells where the internal potential is purely imaginary and the external one real:

$$U(r) = \begin{cases} -iW, & r < R_1\,, \\ -V, & R_1 < r < R_2\,, \\ 0, & r > R_2\,. \end{cases}$$

The scattering length for this potential is given by

$$a_\mathrm{s} = R_2 - \frac{1}{p}\tan\left[p(R_2 - R_1) - \arctan\left\{\frac{p\tan\kappa R_1}{\kappa}\right\}\right].$$

Here $\kappa^2 = 2imW$ and $p^2 = 2mV$.

If the external part of the potential is strong enough to create near-threshold bound or resonant states, one obtains

$$-\mathrm{Im}\ a_\mathrm{s} \approx \frac{1}{\sqrt{2mW}}\frac{1}{\cos^2 p(R_2 - R_1)}\,.$$

These states manifest themselves as an additional enhancement factor $1/\cos^2 p(R_2 - R_1)$.

In the case of an exact resonance (the bound state has an energy exactly equal to zero $p(R_2 - R_1) = (2n+1)\frac{\pi}{2}$) instead of a decreasing function seen previously we obtain an increasing one when $W \to \infty$:

$$-\mathrm{Im}\ a_\mathrm{s} \approx \frac{\sqrt{mW}}{p^2}.$$

The presence of near-threshold poles (bound states or resonances) can significantly change the behaviour of Im a_s.

3 $\bar{\mathrm{p}}$D system

In the case of $\bar{\mathrm{p}}$D scattering, it is possible to obtain numerical solution of the Faddeev equations. In this approach, all three-body effects are taken into account. We performed such a calculation with a simple model of the $\bar{\mathrm{N}}$N interaction to study the effect of strong annihilation and find out whether it is possible to reproduce the experimentally observed tendency. All interactions in this three-body system were chosen in local form to avoid problems related to particular properties of separable interaction.

We solve the Faddeev equations in the coordinate space. The method can be found in [19]. Here we mention only main points.

3.1 Description of the model

The three-body wave function in the Faddeev approach is represented as the sum of three components

$$\Psi = \Psi_1(\mathbf{x}_1, \mathbf{y}_1) + \Psi_2(\mathbf{x}_2, \mathbf{y}_2) + \Psi_3(\mathbf{x}_3, \mathbf{y}_3),$$

where $\mathbf{x}_i, \mathbf{y}_i$ $(i = 1, 2, 3)$ denote three sets of Jacobi coordinates defined according to

$$\mathbf{x}_i = \mathbf{r}_j - \mathbf{r}_k,$$
$$\mathbf{y}_i = \frac{2}{\sqrt{3}}\left(\frac{\mathbf{r}_j + \mathbf{r}_k}{2} - \mathbf{r}_i\right).$$

Here $\mathbf{r}_i$ is nucleon (or antinucleon) position vector.

The three-body Schrödinger equation is then rewritten as the system of equations for the Faddeev components:

$$\begin{aligned}
&\left[k^2 + \Delta_{\mathbf{x}_1} + \Delta_{\mathbf{y}_1} - mV_1(\mathbf{x}_1)\right]\Psi_1(\mathbf{x}_1,\mathbf{y}_1) = \\
&\qquad mV_1(\mathbf{x}_1)[\Psi_2(\mathbf{x}_2,\mathbf{y}_2) + \Psi_3(\mathbf{x}_3,\mathbf{y}_3)], \\
&\left[k^2 + \Delta_{\mathbf{x}_2} + \Delta_{\mathbf{y}_2} - mV_2(\mathbf{x}_2)\right]\Psi_2(\mathbf{x}_2,\mathbf{y}_2) = \\
&\qquad mV_2(\mathbf{x}_2)[\Psi_3(\mathbf{x}_3,\mathbf{y}_3) + \Psi_1(\mathbf{x}_1,\mathbf{y}_1)], \\
&\left[k^2 + \Delta_{\mathbf{x}_3} + \Delta_{\mathbf{y}_3} - mV_3(\mathbf{x}_3)\right]\Psi_3(\mathbf{x}_3,\mathbf{y}_3) = \\
&\qquad mV_3(\mathbf{x}_3)[\Psi_1(\mathbf{x}_1,\mathbf{y}_1) + \Psi_2(\mathbf{x}_2,\mathbf{y}_2)], \qquad (3)
\end{aligned}$$

where E is the 3-body energy (above the mass of the $\bar{\mathrm{N}}$np system) and $k^2 = mE$. To simplify the treatment, we choose the $\bar{\mathrm{N}}$N interaction in the isospin independent form. Therefore we put $V_1(x) = V_2(x) = U(x)$, where U is the complex antinucleon-nucleon potential. In this case the components Ψ_1 and Ψ_2 have the same functional dependence on their arguments. We put also $V_3(x) = V_{\mathrm{d}}(x)$, where $V_{\mathrm{d}}(x)$ is the np potential responsible for describing the deuteron. For $V_{\mathrm{d}}(x)$ we use the Mafliet-Tjon potential [20]. It corresponds to the S-wave np interaction only. Since we consider the low-energy scattering, we keep also only the S-wave in the antinucleon-deuteron scattering. Hence, the Faddeev components depend on the moduli x_i, y_i. Instead of $\Psi_i(x_i, y_i)$ we introduce $\psi_i(x_i, y_i)$:

$$\Psi_i(x_i, y_i) = \frac{\psi_i(x_i, y_i)}{x_i y_i}.$$

In the r.h.-sides of eqs. (3), the moduli of the Jacobi coordinates x_2, y_2 and x_3, y_3 expressed in terms of $\mathbf{x}_1, \mathbf{y}_1$, depend on the angle θ between $\mathbf{x}_1$ and $\mathbf{y}_1$. Therefore the partial-wave decomposition results in the integrals over $u = \cos\theta$. In this way, eqs. (3) turn into the two-component system of integro-differential equations:

$$\begin{aligned}
&\left[k^2 + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - mU(x)\right]\psi_1(x,y) = \\
&\qquad mU(x)\frac{1}{2}\int_{-1}^{+1} \mathrm{d}u \frac{xy}{x'y'}\left[\psi_1(x',y') + \psi_3(x',y')\right], \\
&\left[k^2 + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - mV_{\mathrm{d}}(x)\right]\psi_3(x,y) = \\
&\qquad mV_{\mathrm{d}}(x)\int_{-1}^{+1} \mathrm{d}u \frac{xy}{x'y'}\psi_1(x',y'), \qquad (4)
\end{aligned}$$

with

$$\begin{aligned}
x' &= \frac{1}{2}(x^2 + 2\sqrt{3}xyu + 3y^2)^{1/2}, \\
y' &= \frac{1}{2}(3x^2 - 2\sqrt{3}xyu + y^2)^{1/2}.
\end{aligned}$$

We have taken into account that the dependence of the Faddeev components in the integrands on two sets of variables (*e.g.*, on x_2, y_2 and x_3, y_3 in the first equation in (3)) is reduced, because of integration over u, to the single set x', y'.

Below the three-body threshold, in the asymptotic region of x, y the component $\psi_1(x, y)$ decreases exponentially. The component $\psi_3(x, y)$ describes the system asymptotically (when the distance $\sqrt{3}y/2$ between antiproton and deuteron is large):

$$\psi_1(x, y \to \infty) = \varphi_{\mathrm{d}}(x)\left[\sin(yq) + f(q)\exp(iqy)\right]. \qquad (5)$$

Here $\varphi_{\mathrm{d}}(x)$ is the deuteron wave function, $f(q)$ is the antiproton-deuteron scattering amplitude which is related to the corresponding phase shift δ by $f(q) = (\exp(2i\delta) - 1)/(2i)$. The momentum q is related to the center-of-mass $\bar{\mathrm{p}}$D energy as $E_{\mathrm{cm}}^{\bar{\mathrm{p}}\mathrm{D}} = q^2/m$. With these definitions of $f(q)$ and q the scattering length is found to be

$$a = -\frac{\sqrt{3}}{2}\lim_{q\to 0}\frac{f(q)}{q}.$$

The factor $\sqrt{3}/2$ appears since it enters the relation between the Jacobi coordinate $\mathbf{y}$ and the relative $\bar{\mathrm{p}}$D distance.

The $\bar{\mathrm{N}}$N potential was taken in the Woods-Saxon form to reproduce main features of Kohno-Wiese optical potential [17,18]:

$$\begin{aligned}
U(r) = \; &-V\left[1 + \exp\left\{\frac{r - R_r}{r_r}\right\}\right]^{-1} \\
&-iW\left[1 + \exp\left\{\frac{r - R_i}{r_i}\right\}\right]^{-1},
\end{aligned}$$

with $R_r = 1.2$ fm, $r_r = 0.3$ fm, $R_i = 0.55$ fm, $r_i = 0.2$ fm (to reproduce the fact that the range of the real part is bigger than that of the imaginary part). V and W are considered as parameters. The imaginary part of the interaction is exactly the same as in the original potential [17,18].

3.2 Pure annihilation and resonance limits

The simplest and very instructive case of the $\bar{\mathrm{N}}$N interaction is the limit of pure annihilation $V = 0$. The imaginary part of the scattering length as a function of the imaginary depth W in two ($\bar{\mathrm{N}}$N) and three ($\bar{\mathrm{N}}$D) body problems are presented in fig. 1.

The results coincide with naive geometrical estimations: the imaginary part of the scattering length appears to be higher in the three-body problem. For strong annihilation, the difference between the two values becomes smaller. This result is quite natural when we take into account the arguments given in sect. 2: for large values of W, Im a_{s} does not depend on the size of nucleus R and is determined only by diffuseness.

This limit shows that the observed relation between Im a_{s} for $\bar{\mathrm{p}}$p and $\bar{\mathrm{p}}$D interactions cannot be explained within a model of pure annihilation.

To make Im a_{s} for the $\bar{\mathrm{p}}$p system bigger than for the $\bar{\mathrm{p}}$D system, it is necessary to create a near-threshold resonance or bound state (a pole of the S-matrix) in the two-body system. Figure 2 shows an example when the real

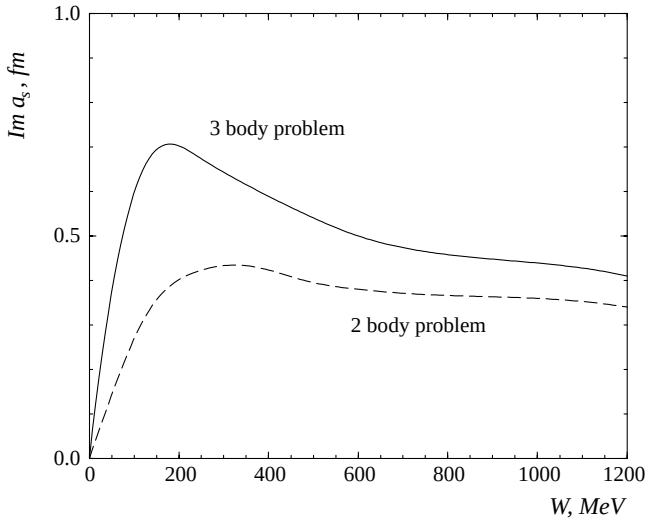

Fig. 1. The imaginary part of the scattering length as a function of the imaginary part W of the Woods-Saxon potential when the two-body $\bar{N}N$ interaction is pure annihilation one (two-body problem: dashed line; three-body problem: solid line).

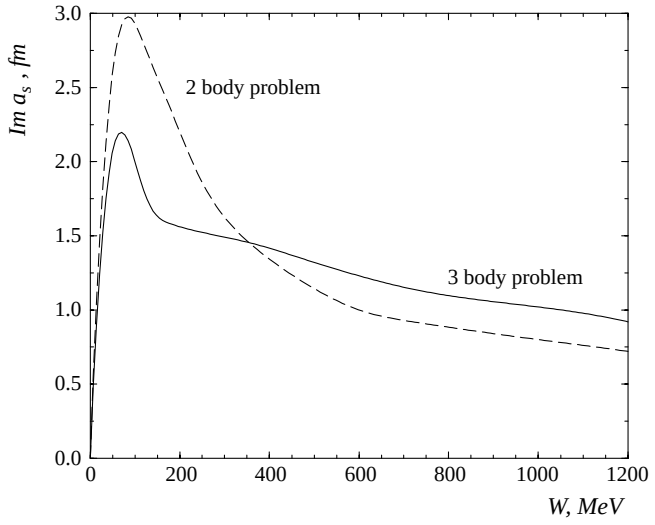

Fig. 2. The imaginary part of the scattering length as a function of the imaginary part W of the Woods-Saxon potential when the two-body $\bar{N}N$ resonances are close to the threshold.

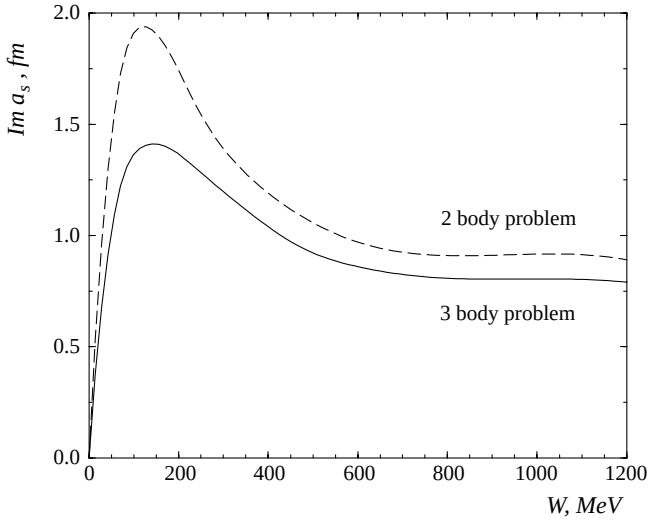

Fig. 3. The imaginary part of the scattering length as a function of the imaginary part W of the Woods-Saxon potential when the two-body $\bar{N}N$ resonances are far from the threshold.

part of the two-body interaction V is strong enough (80 MeV) to be able to produce a S-wave pole which is very close to the threshold. Without annihilation ($W = 0$), one has a loosely bound state with the S-matrix pole position in k-plane of $k_b = i31.7$ MeV/c. When annihilation is switched on this state acquires a width and is shifted away from the threshold: for $W = 50$ MeV, $k_b = -17.3 + i32.2$ MeV/c and for $W = 500$ MeV, $k_b = -130.9 + i72.9$ MeV/c.

This state manifests itself by quite a large value of Im a_s, as we discussed previously, in both two- and three-body problems. However, this resonance is less pronounced in the three-body case and thus Im a_s in this case is less sensitive to variation of the annihilation strength.

This example shows how one can change the relation between Im a_s in two- and three-body problems.

There are, at least, two ways to obtain a more realistic value of the imaginary part of the scattering length. Firstly, one can shift this state quite far from the threshold. Secondly, one can create not ground but an excited state which is sufficiently less sensitive to annihilation because of the zero in the wave function. Figure 3 shows an example of such a situation ($V = 500$ MeV). Without annihilation, the potential creates a loosely bound near threshold state (with $k_b = i36.5$ MeV/c) which is, in fact, the first excited state (the ground state is very deep $k_b = i485$ MeV/c). When the annihilation is switched on, the corresponding pole is shifted out of the threshold as previously but it is less sensitive to annihilation: for $W = 50$ MeV, $k_b = -11 + i37.8$ MeV/c and for $W = 500$ MeV, $k_b = -46 + i80.3$ MeV/c.

It is interesting to note that in Kohno-Weise model [17, 18] such quite large resonances appear in all 4 (!) partial waves (with two different spins and isospins) [10]. Moreover, one of them is an excited state.

As was found in [21] these states appear also in a separable potential which was used for the three-body calculations [5].

4 Conclusions

Two- and three-body systems with very strong annihilation have been investigated using the optical-model potential to understand the behaviour of the imaginary part of the scattering length as a function of the annihilation strength.

In two-body systems, the imaginary part of the scattering length is sensitive mostly to the diffuseness of the potential. This circumstance gives a quite strong experi-

mental prediction: the low-energy antineutron-nucleus annihilation cross-section should be approximately the same for all nuclei.

Due to Coulomb forces, the situation in antiproton-nucleus annihilation can be more complicated: higher partial waves and the real part of the scattering amplitude could modify this simple picture. However, the experiments on the future AD (antiproton decelerator) facility would be of vital importance to understand the antinucleon-nucleus interaction.

For three-body systems, we propose a quite simple mechanism which is able to explain the observation that the imaginary part of the antiproton-deuteron scattering length is smaller than the antiproton-proton one. This mechanism is based on two points. Firstly, the imaginary part of the scattering length in both systems are determined by the diffuseness of the annihilation potential. Thus these values are close to each other. Secondly, the two-body $\bar{\mathrm{N}}\mathrm{N}$ interaction produces S-wave poles and enhances the values of the scattering length. This enhancement is more important in the $\bar{\mathrm{p}}\mathrm{p}$ system than in the $\bar{\mathrm{p}}\mathrm{D}$ one. These S-wave poles (resonances) are necessarily present in all models which describe the experimental data, however, their experimental observation is a delicate task because of their large widths.

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